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Amended Formula for the Decay of Radioactive Material for Cosmic Times

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An amended formula for the decay of radioactive material is presented. It is a modification of the standard exponential formula. The new formula applies for long cosmic times comparable to the Hubble time. It reduces to the standard formula for short times. It is shown that the material decays faster than expected. The application of the new formula to direct measurements of the age of the universe and its implications are briefly discussed.

1. INTRODUCTION

In this paper we present an amended formula for the decay of radioactive material for cosmic times when the times of the decay are of the order of magnitude of the Hubble time. It reduces to the standard formula for short times.

We assume, as usual, that the probability of disintegration during any interval of *cosmic* time dt' is a constant,

$$\frac{dN}{dt'} = -\frac{1}{T'} N \tag{1a}$$

in analogy to the standard formula

$$\frac{dN}{dt} = -\frac{1}{T} N \tag{1b}$$

where T' is a constant to be determined in terms of the half-lifetime T of the decaying material.

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It has been shown that the addition of two cosmic times t_1 backward with respect to us (now) and t_2 backward with respect to t_1 is not just $t_1 + t_2$. Rather, it is given by[1–5]

$$t_{1+2} = \frac{t_1 + t_2}{1 + t_1 t_2/\tau^2} \tag{2}$$

where τ is the Hubble time in the limit of zero gravity, and thus it is a universal constant. Equation (2) is the universal formula for the addition of cosmic times, and reduces to the standard formula of times $t_{1+2} = t_1 + t_2$ for short times with respect to τ.

2. DERIVATION OF THE FORMULA

Let us substitute in formula (2) for the addition of cosmic times $t_1 = -t$ and $t_2 = -dt$. Then

$$t_{1+2} = -\frac{t + dt}{1 + t\,dt/\tau^2} \approx -(t + dt)\left(1 - \frac{t\,dt}{\tau^2}\right) \approx -\left[t + dt\left(1 - \frac{t^2}{\tau^2}\right)\right] \tag{3}$$

Accordingly,

$$-(t + dt) \to -\left[t + dt\left(1 - \frac{t^2}{\tau^2}\right)\right] \tag{4}$$

or

$$-dt \to -dt\left(1 - \frac{t^2}{\tau^2}\right) \tag{5}$$

So far the times denote backward times. Since radioactivity deals with forward times, we use now the standard notation of times, and Eq. (5) will be written as

$$dt \to dt' = dt\left(1 - \frac{t^2}{\tau^2}\right) \tag{6}$$

Equation (1a) will thus have the form

$$\left(1 - \frac{t^2}{\tau^2}\right)^{-1} \frac{dN}{dt} = -\frac{1}{T'} N \tag{7}$$

The solution of Eq. (7) is then given by

$$N = N_0 \exp\left[-\frac{t}{T'}\left(1 - \frac{t^2}{3\tau^2}\right)\right] \tag{8}$$

in analogy to the solution of the standard equation (1b),

$$N = N_0 \exp\left(-\frac{t}{T}\right) \tag{9}$$

3. DETERMINING T' IN TERMS OF HALF-LIFETIME T

From the solution (9) we have

$$N(T) = N_0/e \tag{10}$$

where T is the half-lifetime of the material, as expected. From Eq. (8), we obtain

$$N(T') = N_0 \exp\left[-\left(1 - \frac{T'^2}{3\tau^2}\right)\right] = \frac{N_0}{e} \exp \frac{T'^2}{3\tau^2} \tag{11}$$

Using Eq. (10), we now have

$$N(T') = N(T) \exp \frac{T'^2}{3\tau^2} \tag{12}$$

Under the assumption that $T' \neq 0$, we thus have

$$N(T') > N(T) \tag{13}$$

In order to determine T' in terms of T, we proceed as follows. We substitute in Eq. (8) $t = T$, and using Eq. (10), we obtain

$$N(T) = N_0 \exp\left[-\frac{T}{T'}\left(1 - \frac{T^2}{3\tau^2}\right)\right] = \frac{N_0}{e} \tag{14}$$

As a result we have

$$\frac{T}{T'}\left(1 - \frac{T^2}{3\tau^2}\right) = 1 \tag{15}$$

or

$$T' = T\left(1 - \frac{T^2}{3\tau^2}\right) \tag{16}$$

and thus

$$T' < T \tag{17}$$

Using Eq. (16) in Eq. (8) we therefore obtain

$$N(t) = N_0 \exp\left[-\frac{t(1 - t^2/3\tau^2)}{T(1 - T^2/3\tau^2)}\right] \tag{18}$$

Accordingly we have

$$N(t) = N_0 \exp\left[\frac{-t\alpha(t)}{T}\right] \tag{19}$$

where

$$\alpha(t) = \frac{1 - t^2/3\tau^2}{1 - T^2/3\tau^2} \geq 1 \qquad (t \leq T) \tag{20}$$

Also we have

$$N_0 \exp\left[\frac{-t\alpha(t)}{T}\right] \leq N_0 \exp\left(-\frac{t}{T}\right) \tag{21}$$

Consequently, Eq. (19) provides a large deviation from Eq. (9) when T is comparable to τ and we measure radioactivity over astronomical times. For example, thorium is a radioactive element with a half-life of 14.1 billion years, as compared to the estimated 13 billion-year age of the universe. Such measurements/observations can be carried out, and the detected deviations can be drawn by a graph (see Fig. 1). In principle, it follows from Eq. (19) that $N(t)$ for a given t is less than that obtained through the traditional formula; i.e., the material decays faster than expected.

4. DISCUSSION

Accurate measurements for the decay of radioactive materials from the earth and from stars in our galaxy could provide crucial information about the age of the universe. It is well known that two of the most straightforward methods of calculating the age of the universe—through redshift measurements and through stellar evolution—yield incompatible results. Recent measurements of the distances of faraway galaxies through the use of the Hubble Space Telescope indicate an age much less than the ages of the oldest stars that we calculate through the stellar evolution theory.[6–16]

At present there is no conclusion to this problem; a cosmological constant would probably clarify the situation, but it is possible that the discrepancy

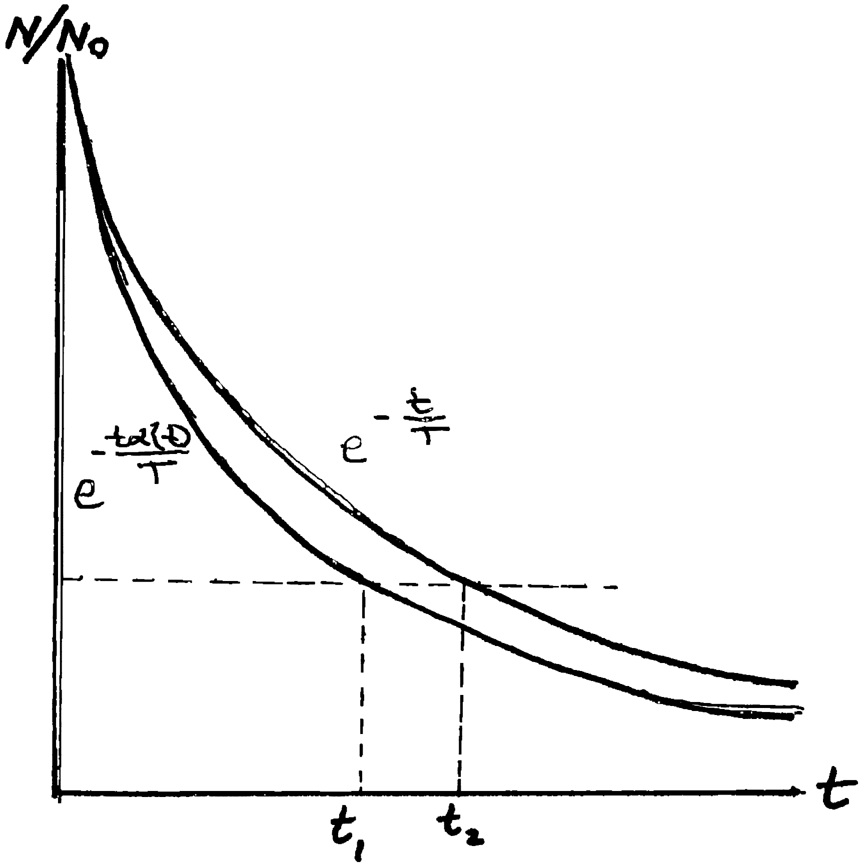

Fig. 1. Two curves describing the standard exponential decay $N/N_0 = \exp(-t/T)$ and the amended cosmic decay $N/N_0 = \exp[-t\alpha(t)/T]$. For a measured N/N_0 the two curves indicate two different times t_1 and t_2, with $t_2 > t_1$, where t_1 and t_2 correspond to the amended and the standard decay formulas. Accordingly, cosmic times of decaying materials on earth and stars are actually shorter than has been believed so far.

will disappear with more accurate measurements of the age of the universe using both methods. The discussion given in this paper clearly goes in the right direction in solving this important impasse.

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